Dynamic realization of transport phenomenon in finite system

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Exploring the Fokker-Planck or Langevin type transport equation from the underlying dynamics, we study by exploiting the numerical simulations whether or not one may expect dissipative motions in a Hamilton system with finite degrees of freedom without introducing any statistical concept. It will be shown that the macro-level transport phenomenon in a finite system can be *dynamically* established from the underlying Hamilton system, and the nonlinear coupling between two subsystems is decisive for generating the transport phenomenon.

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I. INTRODUCTION

The Fokker-Planck or Langevin type equations have been widely applied to the macroscopic averaged motion in the vast fields of contemporary science. The evolution of the early universe, that of the chemical reaction, many active processes in the biological system, the fission and fusion processes in the nuclear system, and the measurement theory are typical examples among others. Although these processes have been described successfully by the phenomenological transport equation, there still remain some basic problems, such as how to derive it from the fundamental level dynamics, how the statistical state is realized in the irrelevant subsystem, and why the irreversible macro-level process is generated as a result of the reversible micro-level dynamics. Several papers on this subject [1–15] have been published.

In many approaches [10,12,13], however, a division between the relevant and irrelevant degrees of freedom is made by hand, and the relevant system is assumed to be *linearly* coupled with the irrelevant reservoir. As is well known, the theoretical difficulties in deriving the irreversible process are drastically reduced, when the linear response theory (LRT) is adopted together with the irrelevant reservoir. From the macroscopic point of view, such a statistical approach seems to be reasonable for the infinite system. In such an isolated finite many-body quantum system as the nucleus where assumption of a large number of degrees of freedom is not justified, and in a case when one intends to derive the phenomenological transport equation from the fundamental level dynamics, it is not obvious whether or not one may introduce the reservoir for the irrelevant degrees of freedom, and whether the total system is divided into the relevant and irrelevant degrees of freedom by leaving a resultant *linear* coupling between them.

To get a full understanding of the dynamical realization of the statistical state in a finite system, in our previous paper [8] an evolution process of a simple two-degrees of freedom system has been studied by using a general microscopic transport theory [7,9,16]. It has been shown that the nonlinear coupling between different degrees of freedom responsible for generating a chaotic motion plays an important role in realizing a statistical state for a system described by a bundle of trajectories. It has also been pointed out that the response function shows quite different behavior from the well known linear coupling case, when two systems are coupled with a nonlinear interaction. The above system with two degrees of freedom is too simple to assign the relevant degree of freedom or to discuss its dissipation, because the chaotic or statistical state can be realized by a system with at least two degrees of freedom. In order to study the dissipation process microscopically, it is inevitable to treat a system with more than two degrees of freedom, which is able to be divided into two weakly coupled subsystems: one is composed of at least two degrees of freedom and is regarded as an irrelevant system, whereas the rest are considered to be a relevant system.

The main objective of this paper is to study one step further how and why the dissipative process phenomenologically described by the Langevin type equation is realized as a result of the underlying dynamics, what kinds of necessary conditions there are in realizing the dissipative process, what kinds of dynamical relations there are between the micro-level and phenomenological-level descriptions, and whether the linear and nonlinear couplings between the relevant and irrelevant systems generate some substantial differences in the dissipation mechanism, by using specific model Hamiltonians. In Sec. II, we briefly recapitulate the theory of the nuclear coupled-master equation [9] for the sake of selfcontainment. Starting from the most general coupled-master equation, we try to derive the Fokker-Planck and Langevin type equation, by clarifying necessary underlying conditions. Our purpose is to realize such a physical situation where these conditions are satisfied; in Sect. III, various numerical simulations will be performed for a system where a relevant (collective) harmonic oscillator is coupled with the irrelevant (intrinsic) SU(3) model. After numerically realizing a macro-level transport phenomena, we will try to reproduce it by using a phenomenological Langevin equation, whose potential is derived microscopically. By using the β Fermi-Pasta-Ulam (β-FPU) model Hamiltonian, we will further explore how different transport phenomena will appear when the two systems are coupled with linear or nonlinear interactions. Sec. IV is devoted to a summary and discussions.

II. TRANSPORT EQUATION OF COLLECTIVE MOTION

A. Nuclear coupled master equation

Exploring the microscopic theory of nuclear large-amplitude collective dissipative motion, whose characteristic energy per nucleon is much smaller than the Fermi energy, one may start with the time-dependent Hartree-Fock (TDHF) theory. Since the basic equation of the TDHF theory is known to be formally equivalent to the classical canonical equations of motion [17], the use of the TDHF theory enables us to investigate the basic ingredients of the nonlinear nuclear dynamics in terms of the TDHF trajectories. The TDHF equation is expressed as

$$\delta\langle\Phi(t)|\left(i\frac{\partial}{\partial t}-\hat{H}\right)|\Phi(t)\rangle=0, \tag{1}$$

where $|\Phi(t)\rangle$ is the general time-dependent single Slater determinant given by

$$|\Phi(t)\rangle=\exp\{i\hat{F}\}|\Phi_0>e^{iE_0t},$$

$$i\hat{F}=\sum_{\mu i}\{f_{\mu i}(t)\hat{a}_\mu^\dagger\hat{b}_i^\dagger-f_{\mu i}^*(t)\hat{b}_i\hat{a}_\mu\}, \tag{2}$$

where $|\Phi_0\rangle$ denotes a HF stationary state, and $\hat{a}_\mu^\dagger(\mu=1,2,\ldots,m)$ and $\hat{b}_i^\dagger(i=1,2,\ldots,n)$ mean the particle- and hole-creation operators with respect to $|\Phi_0\rangle$. The HF Hamiltonian H and the HF energy E_0 are defined as

$$H=\langle\Phi(t)|\hat{H}|\Phi(t)\rangle-E_0, \quad E_0=\langle\Phi_0|\hat{H}|\Phi_0\rangle. \tag{3}$$

With the aid of the self-consistent collective coordinate (SCC) method [18], the whole system can be optimally divided into the relevant (collective) and irrelevant (intrinsic) degrees of freedom by introducing an optimal canonical coordinate system called the dynamical canonical coordinate (DCC) system for a given trajectory. That is, the total closed system $\eta\oplus\xi$ is dynamically divided into two subsystems η and ξ, whose optimal coordinate systems are expressed as η_a, $\eta_a^*:a=1,\cdots$ and ξ_α, $\xi_\alpha^*:\alpha=1,\cdots$, respectively. The resulting Hamiltonian in the DCC system is expressed as

$$H=H_\eta+H_\xi+H_{\rm coupl}, \tag{4}$$

where H_η depends on the relevant, H_ξ on the irrelevant, and $H_{\rm coupl}$ on both the relevant and irrelevant variables. The TDHF equation (1) can then be formally expressed as a set of canonical equations of motion in the classical mechanics in the TDHF phase space (symplectic manifold) as

$$i\dot{\eta}_a=\frac{\partial H}{\partial\eta_a^*}, \quad i\dot{\eta}_a^*=-\frac{\partial H}{\partial\eta_a}, \quad i\dot{\xi}_\alpha=\frac{\partial H}{\partial\xi_\alpha^*}, \quad i\dot{\xi}_\alpha^*=-\frac{\partial H}{\partial\xi_\alpha}. \tag{5}$$

Here, it is worth noting that the SCC method defines the DCC system in order to eliminate the *linear* coupling between the relevant and irrelevant subsystems, i.e., the maximal decoupling condition [9] given by

$$\left.\frac{\partial H_{\rm coupl}}{\partial\eta}\right|_{\xi=\xi^*=0}=0 \tag{6}$$

is satisfied. This separation in the degrees of freedom will turn out to be very important for exploring the energy dissipation process and nonlinear dynamics between the collective and intrinsic modes of motion.

The transport, dissipative, and damping phenomena appearing in the nuclear system may involve dynamics described by the wave packet rather than that by the eigenstate. Within the mean-field approximation, these phenomena may be expressed by the collective behavior of the ensemble of TDHF trajectories, rather than the single trajectory. A difference between the dynamics described by the single trajectory and by the bundle of trajectories might be related to the controversy on the effects of one-body and two-body dissipations.

To deal with the ensemble of TDHF trajectories, we start with the Liouville equation for the distribution function

$$\dot{\rho}(t)=-i\mathcal{L}\rho(t), \quad \mathcal{L}^*\equiv i\{H,^*\}_{\rm PB},$$

$$\rho(t)=\rho(\eta(t),\eta(t)^*,\xi(t),\xi(t)^*), \tag{7}$$

which is equivalent to TDHF equation (1). Here the symbol $\{\}_{\rm PB}$ denotes the Poisson bracket. Since we are interested in the time evolution of the bundle of TDHF trajectories, whose bulk properties ought to be expressed by the relevant variables alone, we introduce the reduced distribution functions as

$$\rho_\eta(t)={\rm Tr}_\xi\,\rho(t), \quad \rho_\xi(t)={\rm Tr}_\eta\,\rho(t). \tag{8}$$

Here, the total distribution function $\rho(t)$ is normalized so as to satisfy the relation

$${\rm Tr}\,\rho(t)=1, \tag{9}$$

where

$${\rm Tr}\equiv{\rm Tr}_\eta\,{\rm Tr}_\xi, \quad {\rm Tr}_\eta\equiv\prod_a\int\int d\eta_a\,d\eta_a^*,$$

$${\rm Tr}_\xi\equiv\prod_\alpha\int\int d\xi_\alpha\,d\xi_\alpha^*. \tag{10}$$

With the aid of the reduced distribution functions $\rho_\eta(t)$ and $\rho_\xi(t)$, one may decompose the Hamiltonian in Eq. (4) into the form

$$\begin{aligned}H&=H_\eta+H_\xi+H_{\rm coupl}\\&=H_\eta+H_\eta(t)+H_\xi+H_\xi(t)+H_\Delta(t)-E_0(t),\\H_\eta(t)&\equiv{\rm Tr}_\xi\,H_{\rm coupl}\rho_\xi(t), \quad H_\xi(t)\equiv{\rm Tr}_\eta\,H_{\rm coupl}\rho_\eta(t),\\H_{\rm aver}(t)&\equiv H_\eta(t)+H_\xi(t), \quad E_0(t)\equiv{\rm Tr}\,H_{\rm coupl}\rho(t),\\H_\Delta(t)&\equiv H_{\rm coupl}-H_{\rm aver}(t)+E_0(t).\end{aligned} \tag{11}$$

By exploiting the time-dependent projection operator method [19], one may decompose the distribution function into a separable part and a correlated one as

$$\rho(t)=\rho_s(t)+\rho_c(t),$$

$$\rho_s(t)\equiv P(t)\rho(t)=\rho_\eta(t)\rho_\xi(t), \quad \rho_c(t)\equiv(1-P(t))\rho(t), \tag{12}$$

where $P(t)$ is the time-dependent projection operator defined by

$$P(t)\equiv\rho_\eta(t)\mathrm{Tr}_\eta+\rho_\xi(t)\mathrm{Tr}_\xi-\rho_\eta(t)\rho_\xi(t)\mathrm{Tr}_\eta\,\mathrm{Tr}_\xi. \tag{13}$$

From the Liouville equation (7), one gets

$$\dot{\rho}_s(t)=-iP(t)\mathcal{L}\rho_s(t)-iP(t)\mathcal{L}\rho_c(t), \tag{14}$$

$$\dot{\rho}_c(t)=-i(1-P(t))\mathcal{L}\rho_s(t)-i(1-P(t))\mathcal{L}\rho_c(t). \tag{15}$$

By introducing

$$g(t,t')\equiv T\exp\left\{-i\int_{t'}^{t}[1-P(\tau)]\mathcal{L}d\tau\right\}, \tag{16}$$

where T denotes the time ordering operator, one obtains the master equation for $\rho_s(t)$ as

$$\dot{\rho}_s(t)=-iP(t)\mathcal{L}\rho_s(t)-iP(t)\mathcal{L}g(t,t_I)\rho_c(t_I)-\int_{t_I}^{t}dt'P(t)\mathcal{L}g(t,t')\{1-P(t')\}\mathcal{L}\rho_s(t'), \tag{17}$$

where t_I stands for an initial time. As is easily proved, the Liouvillian $\mathcal{L}$ appearing inside the time integration in Eq. (17) is replaced by $\mathcal{L}_{\mathrm{coupl}}$ defined by $\mathcal{L}_{\mathrm{coupl}}{*}=\{H_{\mathrm{coupl}},*\}_{\mathrm{PB}}$. Expressing $\rho_s(t)$ and $P(t)$ in terms of $\rho_\eta(t)$ and $\rho_\xi(t)$, and operating Tr_η and Tr_ξ in Eq. (17), one obtains a coupled master equation

$$\dot{\rho}_\eta(t)=-i[\mathcal{L}_\eta+\mathcal{L}_\eta(t)]\rho_\eta(t)-i\,\mathrm{Tr}_\xi[\mathcal{L}_\eta+\mathcal{L}_{\mathrm{coupl}}]g(t,t_I)\rho_c(t_I)-\int_{t_I}^{t}d\tau\,\mathrm{Tr}_\xi\,\mathcal{L}_\Delta(t)g(t,\tau)\mathcal{L}_\Delta(\tau)\rho_\eta(\tau)\rho_\xi(\tau), \tag{18}$$

$$\dot{\rho}_\xi(t)=-i[\mathcal{L}_\xi+\mathcal{L}_\xi(t)]\rho_\xi(t)-i\,\mathrm{Tr}_\eta[\mathcal{L}_\xi+\mathcal{L}_{\mathrm{coupl}}]g(t,t_I)\rho_c(t_I)-\int_{t_I}^{t}d\tau\,\mathrm{Tr}_\eta\,\mathcal{L}_\Delta(t)g(t,\tau)\mathcal{L}_\Delta(\tau)\rho_\eta(\tau)\rho_\xi(\tau),$$

where $\mathcal{L}_\Delta(t){*}\equiv\{H_\Delta(t),*\}_{\mathrm{PB}}$. The coupled master equation (18) is still equivalent to the original Liouville equation (7) and is not yet tractable.

B. Dynamical response and correlation functions

As was discussed in Ref. [8], a bundle of trajectories even in the two degrees of freedom system may reach a statistical object. In this case, it is reasonable to assume that the effects on the relevant system coming from the irrelevant one are mainly expressed by an averaged effect over the irrelevant distribution function (*Assumption*). Namely, the effects due to the fluctuation part $H_\Delta(t)$ are assumed to be much smaller than those coming from $H_{\mathrm{aver}}(t)$. Under this assumption, one may introduce the *mean-field* propagator

$$g_{\mathrm{mf}}(t,t')=T\exp\left\{-i\int_{t'}^{t}[1-P(\tau)]\mathcal{L}^{\mathrm{mf}}(\tau)d\tau\right\},$$

$$\mathcal{L}^{\mathrm{mf}}(t)=\mathcal{L}_\eta^{\mathrm{mf}}(t)+\mathcal{L}_\xi^{\mathrm{mf}}(t), \quad \mathcal{L}_\eta^{\mathrm{mf}}(t)\equiv\mathcal{L}_\eta+\mathcal{L}_\eta(t),$$

$$\mathcal{L}_\xi^{\mathrm{mf}}(t)\equiv\mathcal{L}_\xi+\mathcal{L}_\xi(t), \tag{19}$$

which describes the major time evolution of the system, while the fluctuation part is regarded as a perturbation. By further introducing the following propagators given by

$$G_{\mathrm{mf}}(t,t')\equiv T\exp\left\{-i\int_{t'}^{t}\mathcal{L}^{\mathrm{mf}}(\tau)d\tau\right\}=G_\eta(t,t')G_\xi(t,t'),$$

$$G_\eta(t,t')\equiv T\exp\left\{-i\int_{t'}^{t}\mathcal{L}_\eta^{\mathrm{mf}}(\tau)d\tau\right\},$$

$$G_\xi(t,t')\equiv T\exp\left\{-i\int_{t'}^{t}\mathcal{L}_\xi^{\mathrm{mf}}(\tau)d\tau\right\}, \tag{20}$$

one may prove that there is a relation

$$g_{\mathrm{mf}}(t,\tau)\mathcal{L}_\Delta(\tau)\rho_\eta(\tau)\rho_\xi(\tau)=G_{\mathrm{mf}}(t,\tau)\mathcal{L}_\Delta(\tau)\rho_\eta(\tau)\rho_\xi(\tau). \tag{21}$$

The coupling interaction is generally expressed as

$$H_{\mathrm{coupl}}(\eta,\xi)=\sum_l A^l(\eta)B^l(\xi). \tag{22}$$

For simplicity, we hereafter discard the summation l in the coupling. By introducing the generalized two-time correlation and response functions, which have been called *dynamical* correlation and response functions in Ref. [7], through

$$\phi(t,\tau)\equiv\mathrm{Tr}_\xi\,G_\xi(\tau,t)B(B-\langle B\rangle_t)\rho_\xi(\tau), \tag{23}$$

$$\chi(t,\tau)\equiv\mathrm{Tr}_\xi\{G_\xi(\tau,t)B,B\}_{\mathrm{PB}}\rho_\xi(\tau), \tag{24}$$

with $\langle B\rangle_t\equiv Tr_\xi B\rho_\xi(t)$, the master equation in Eq.(18) for the relevant degree of freedom is expressed as

$$\dot{\rho}_\eta(t)=-i[\mathcal{L}_\eta+\mathcal{L}_\eta(t)]\rho_\eta(t)-i\,\mathrm{Tr}_\xi[\mathcal{L}_\eta+\mathcal{L}_{\mathrm{coupl}}]g(t,t_I)\times\rho_c(t_I)+\int_0^{t-t_I}d\tau\chi(t,t-\tau)\{A,G_\eta(t,t-\tau)\times(A-\langle A\rangle_{t-\tau})\rho_\eta(t-\tau)\}_{\mathrm{PB}}+\int_0^{t-t_I}d\tau\phi(t,t-\tau)\times\{A,G_\eta(t,t-\tau)\{A,\rho_\eta(t-\tau)\}_{\mathrm{PB}}\}_{\mathrm{PB}}, \tag{25}$$

with $\langle A\rangle_t \equiv \mathrm{Tr}_\eta A\rho_\eta(t)$. Here, it should be noted that the whole system is developed exactly up to t_I. In order to make Eq. (25) applicable, t_I should be taken to be very close to a time when the irrelevant system approaches very near to its stationary state (i.e., the irrelevant system is very near to the statistical state where one may safely make the assumption to be stated in next Sec. II C). In order to analyze what happens in the microscopic system, which is situated far from its stationary states, one has to study $\chi(t_I,t_I-\tau)$ and $\phi(t_I,t_I-\tau)$ by changing t_I. Since both $\chi(t_I,t_I-\tau)$ and $\phi(t_I,t_I-\tau)$ are strongly dependent on t_I, it is not easy to explore the dynamical evolution of the system far from the stationary state. Therefore, to make Eq. (25) applicable, we will exploit the further assumptions.

C. Macroscopic transport equation

In this subsection, we discuss how the macroscopic transport equation is obtained from the fully microscopic master equation (25) by clearly itemizing necessary microscopic conditions.

Condition I. Suppose the relevant distribution function $\rho_\eta(t-\tau)$ inside the time integration in Eq. (25) evolves through the mean-field Hamiltonian $H_\eta+H_\eta(t)$[1]. Namely, $\rho_\eta(t-\tau)$ inside the integration is assumed to be expressed as $\rho_\eta(t)=G_\eta(t,t-\tau)\rho_\eta(t-\tau)$, so that Eq. (25) is reduced to

$$\begin{aligned}\dot\rho_\eta(t)=&-i[\mathcal{L}_\eta+\mathcal{L}_\eta(t)]\rho_\eta(t)-i\ \mathrm{Tr}_\xi[\mathcal{L}_\eta+\mathcal{L}_{\mathrm{coupl}}]g(t,t_I)\\&\times\rho_c(t_I)+\int_0^{t-t_I}d\tau\chi(t,t-\tau)\{A,G_\eta(t,t-\tau)\\&\times(A-\langle A\rangle_{t-\tau})\cdot\rho_\eta(t)\}_{\mathrm{PB}}+\int_0^{t-t_I}d\tau\phi(t,t\\&-\tau)\{A,\{G_\eta(t,t-\tau)A,\rho_\eta(t)\}_{\mathrm{PB}}\}_{\mathrm{PB}}.\end{aligned}\tag{26}$$

This condition is equivalent to the *Assumption* discussed in the previous subsection, because the fluctuation effects are sufficiently small and are able to be treated as a perturbation around the path generated by the mean-field Hamiltonian $H_\eta+H_\eta(t)$, and are sufficient to be retained in Eq. (26) up to the second order.

Condition II. Suppose the irrelevant distribution function $\rho_\xi(t)$ has already reached its time-independent stationary state $\rho_\xi(t_0)$. According to our previous paper [8], this situation is able to be realized even in the two-degrees of freedom system. Under this assumption, the relevant mean-field Liouvillian $\mathcal{L}_\eta+\mathcal{L}_\eta(t)$ becomes a time independent object. Under the same assumption, a time ordered integration in $G_\eta(t,t')$ defined in Eq. (20) is performed and one may introduce

$$G_\eta(t,t-\tau)\approx G_\eta(\tau)\equiv\exp\{-i\mathcal{L}_\eta^{\mathrm{mf}}\tau\},\quad \mathcal{L}_\eta^{\mathrm{mf}}\equiv\mathcal{L}_\eta+\mathcal{L}_\eta(t_0),\tag{27}$$

where t_0 denotes a time when the irrelevant system has reached its stationary state.

Condition III. Suppose the irrelevant time scale is much shorter than the relevant time scale. Under this assumption, the response $\chi(t,t-\tau)$ and correlation functions $\phi(t,t-\tau)$ are regarded as being independent of the time t, because t in Eq. (26) is regarded as describing a very slow time evolution of the relevant motion. By introducing an approximate one-time response and correlation functions

$$\chi(\tau)\approx\chi(t,t-\tau),\quad \phi(\tau)\approx\phi(t,t-\tau),\tag{28}$$

one may get

$$\begin{aligned}\dot\rho_\eta(t)=&-i[\mathcal{L}_\eta+\mathcal{L}_\eta(t)]\rho_\eta(t)-i\ \mathrm{Tr}_\xi[\mathcal{L}_\eta+\mathcal{L}_{\mathrm{coupl}}]g(t,t_I)\\&\times\rho_c(t_I)+\int_0^\infty d\tau\chi(\tau)\{A,\exp(-i\mathcal{L}_\eta^{\mathrm{mf}}\tau)\\&\times(A-\langle A\rangle_{t-\tau})\rho_\eta(t)\}_{\mathrm{PB}}+\int_0^\infty d\tau\phi(\tau)\\&\times\{A,\{\exp(-i\mathcal{L}_\eta^{\mathrm{mf}}\tau)A,\rho_\eta(t)\}_{\mathrm{PB}}\}_{\mathrm{PB}}.\end{aligned}\tag{29}$$

Here it should be noted that such one-time response and correlation functions are still different from the usual ones introduced in the LRT where the concepts of linear coupling and of heat bath are adopted. Under the same assumption, the upper limit of the integration $t-t_I$ in Eq. (29) can be extended to the infinity, because the $\chi(\tau)$ and $\phi(\tau)$ are assumed to be very fast damping functions when it is measured in the relevant time scale.

Here, one may introduce the susceptibility $\zeta(t)$

$$\zeta(t)=\int_0^t d\tau\chi(\tau),\quad \zeta(0)=0.\tag{30}$$

Defining $\zeta\equiv\zeta(\infty)$, one may further introduce another dynamical function $c(t)$:

$$\zeta(t)=[1-c(t)]\zeta,\quad \text{with } c(0)=1,\quad c(\infty)=0,\tag{31}$$

which satisfies the following relation:

$$\chi(t)=\frac{\partial\zeta(t)}{\partial t}=-\zeta\frac{\partial c(t)}{\partial t}.\tag{32}$$

Inserting Eq. (32) into Eq. (29) and integrating by part, one gets

$$\begin{aligned}\dot\rho_\eta(t)=&-i[\mathcal{L}_\eta+\mathcal{L}_\eta(t)]\rho_\eta(t)-i\,\mathrm{Tr}_\xi[\mathcal{L}_\eta+\mathcal{L}_{\mathrm{coupl}}]g(t,t_I)\\&\times\rho_c(t_I)+\zeta\{A,(A-\langle A\rangle_t)\rho_\eta(t)\}_{\mathrm{PB}}+\zeta\int_0^\infty d\tau c(\tau)\\&\times\left\{A,\frac{d}{d\tau}[\exp(-i\mathcal{L}_\eta^{\mathrm{mf}}\tau)(A-\langle A\rangle_t)]\rho_\eta(t)\right\}_{\mathrm{PB}}\\&+\int_0^\infty d\tau\phi(\tau)\{A,\{\exp(-i\mathcal{L}_\eta^{\mathrm{mf}}\tau)A,\rho_\eta(t)\}_{\mathrm{PB}}\}_{\mathrm{PB}}.\end{aligned}\tag{33}$$

[1]The same assumption has been introduced in a case of the linear coupling [13].

This equation is a Fokker-Planck type equation. The first term on the right-hand side of Eq. (33) represents the contributions from the mean-field part, and the second term a contribution from the correlated part of the distribution function at time t_I. The last three terms represent contributions from the dynamical fluctuation effects H_Δ. The friction as well as fluctuation terms are supposed to emerge as a result of those three terms. We will discuss the role of each term with our numerical simulation in Sec. III.

At the end of this subsection, let us discuss how to obtain the Langevin equation from our fully microscopic coupled master equation, because it has been regarded as a final goal of the microscopic or dynamical approaches to justify the phenomenological approaches. For the sake of simplicity, let us discuss a case where the interaction between relevant and irrelevant degrees of freedom has the following linear form:

$$H_{\rm coupl}=\lambda Q\sum_i q_i,$$

i.e.,

$$A=\sqrt{\lambda}Q, \quad B=\sqrt{\lambda}\sum_i q_i,$$

$$Q=\frac{1}{\sqrt{2}}(\eta+\eta^*), \quad P=\frac{i}{\sqrt{2}}(\eta^*-\eta),$$

$$q_i=\frac{1}{\sqrt{2}}(\xi_i+\xi_i^*), \quad p_i=\frac{i}{\sqrt{2}}(\xi_i^*-\xi_i). \tag{34}$$

Here we assume that the relevant system consists of one degree of freedom described by P,Q. Even though we apply the linear coupling form, the generalization for the case with more general nonlinear coupling is straightforward. In order to evaluate Eq. (33), one has to calculate

$$Q(\tau)=\exp(-i\mathcal{L}_\eta^{\rm mf}\tau)Q, \tag{35}$$

where $Q(\tau)$ is a phase space image of Q through the backward evolution. Thus the Poisson bracket $\{Q(\tau),\rho_\eta(t)\}_{\rm PB}$ in Eq. (33) is expressed as

$$\{Q(\tau),\rho_\eta(t)\}_{\rm PB}=\frac{\partial Q(\tau)}{\partial Q}\frac{\partial\rho_\eta(t)}{\partial P}-\frac{\partial Q(\tau)}{\partial P}\frac{\partial\rho_\eta(t)}{\partial Q}. \tag{36}$$

By introducing the following quantities:

$$\alpha_1(P,Q)\equiv\lambda\int_0^\infty d\tau\phi(\tau)\frac{\partial Q(\tau)}{\partial Q},$$

$$\alpha_2(P,Q)\equiv-\lambda\int_0^\infty d\tau\phi(\tau)\frac{\partial Q(\tau)}{\partial P},$$

$$\beta(P,Q)\equiv\lambda\zeta\int_0^\infty d\tau c(\tau)\frac{\partial Q(\tau)}{\partial\tau}, \tag{37}$$

Eq. (33) is reduced to

$$\begin{aligned}\dot\rho_\eta(t)=&-i\,{\rm Tr}_\xi[\mathcal{L}_\eta+\mathcal{L}_{\rm coupl}]g(t,t_I)\rho_c(t_I)\\&+\Big\{-i(\mathcal{L}_\eta+\mathcal{L}_\eta(t))+\lambda\zeta(Q-\langle Q\rangle_t)\frac{\partial}{\partial P}\\&+\frac{\partial}{\partial P}\beta(P,Q)+\frac{\partial}{\partial P}\alpha_1(P,Q)\frac{\partial}{\partial P}\\&+\frac{\partial}{\partial P}\alpha_2(P,Q)\frac{\partial}{\partial Q}\Big\}\rho_\eta(t).\end{aligned} \tag{38}$$

As discussed in Ref. [12], Eq. (38) results in the Langevin equation with a form

$$\ddot Q=-\frac{1}{m}\frac{\partial U(Q)}{\partial x}-\gamma\dot Q+f(t) \tag{39}$$

by introducing a concept of mechanical temperature. The above derivation of the Langevin equation is still too formal to be applicable for the general cases. However it might be naturally expected that Conditions I, II, and III are met in the actual dynamical processes.

III. DYNAMICAL SIMULATION ON NONLINEAR NUCLEAR SYSTEM

A. Microscopic model

The system considered in our numerical calculation is composed of a collective degree of freedom coupled to intrinsic degrees of freedom through weak interaction, which simulates a nuclear system. The collective system describing, e.g., the giant resonance is represented by the harmonic oscillator given by

$$H_\eta(q,p)=\frac{p^2}{2M}+\frac{1}{2}M\omega^2q^2 \tag{40}$$

and the intrinsic system mimicking the hot nucleus is described by the modified SU(3) model Hamiltonian [20] given by

$$\hat H=\sum_{i=0}^{2}\epsilon_i\hat K_{ii}+\frac{1}{2}\sum_{i=1}^{2}V_i\{\hat K_{i0}\hat K_{i0}+{\rm H.c.}\};$$

$$\hat K_{ij}=\sum_{m=1}^{N}C_{im}^\dagger C_{jm}, \tag{41}$$

where $C_{im}^\dagger$ and C_{im} represent the fermion creation and annihilation operators. There are three N-fold degenerate levels with $\epsilon_0<\epsilon_1<\epsilon_2$. In the case with an even N particle system, the TDHF theory gives a classical Hamiltonian with two degrees of freedom as

$$
\begin{aligned}
&H_{\xi}(q_1,p_1,q_2,p_2)\\
&\quad=\frac{1}{2}(\epsilon_1-\epsilon_0)(q_1^2+p_1^2)+\frac{1}{2}V_1(N-1)(q_1^2-p_1^2)\\
&\qquad-\frac{N-1}{4N}V_1(q_1^4-p_1^4)+\frac{1}{2}(\epsilon_2-\epsilon_0)(q_2^2+p_2^2)\\
&\qquad+\frac{1}{2}V_2(N-1)(q_2^2-p_2^2)-\frac{N-1}{4N}V_2(q_2^4-p_2^4)\\
&\qquad+\frac{N-1}{4N}[-V_1(q_1^2-p_1^2)(q_2^2+p_2^2)\\
&\qquad-V_2(q_1^2+p_1^2)(q_2^2-p_2^2)]. \qquad (42)
\end{aligned}
$$

In our numerical calculation, the parameters used are $M=18.75$, $\omega^2=0.0064$, $\epsilon_0=0$, $\epsilon_1=1$, $\epsilon_2=2$, $N=30$, and $V_i=-0.07$. In this case, the collective time scale $\tau_{\rm col}$ characterized by the harmonic oscillator in Eq. (40) and the intrinsic time scale $\tau_{\rm in}$ characterized by the harmonic part of the intrinsic Hamiltonian in Eq.(42) satisfies a relation $\tau_{\rm col}\sim 10\tau_{\rm in}$.

For the coupling interaction, we use the following nonlinear interaction given by

$$
H_{\rm coupl}=\lambda(q-q_0)^2\sum_{i=1}^{2}\{q_i^2+p_i^2\}. \qquad (43)
$$

A physical meaning of introducing a quantity q_0 in Eq. (43) will be discussed at the end of this subsection as well as the next subsection.

In performing the numerical simulation, the time evolution of the distribution function $\rho(t)$ is evaluated by using the pseudo-particle method as

$$
\begin{aligned}
\rho(t)=&\frac{1}{N_p}\sum_{n=1}^{N_p}\prod_{i=1}^{2}\delta(q_i-q_{i,n}(t))\\
&\times\delta(p_i-p_{i,n}(t))\delta(q-q_n(t))\delta(p-p_n(t)), \qquad (44)
\end{aligned}
$$

where N_p means the total number of pseudoparticles. The collective coordinates $q_n(t)$ and $p_n(t)$, and the intrinsic coordinates $q_{i,n}(t)$ and $p_{i,n}(t)$ determine a phase space point of the nth pseudo-particle at time t, whose time dependence is described by the canonical equations of motion given by

$$
\dot{q}_i=\frac{\partial H}{\partial p_i},\quad \dot{p}_i=-\frac{\partial H}{\partial q_i},\quad \dot{q}=\frac{\partial H}{\partial p},\quad \dot{p}=-\frac{\partial H}{\partial q},
$$

$$
H\equiv H_{\eta}(q,p)+H_{\xi}(q_1,p_1,q_2,p_2)+H_{\rm coupl}. \qquad (45)
$$

We use the fourth order Runge-Kutta method for integrating the canonical equations of motion and N_p is chosen to be 10 000. The initial condition for the intrinsic distribution function is given by a uniform distribution in a tiny region of the stochastic sea as stated in Ref. [8]. That for the collective distribution function is given by the δ function centered at $q(0)=0$ and $p(0)$, $p(0)$ being defined by a given collective energy E_{η} together with $q(0)=0$. The distribution function in Eq. (44) defines an ensemble of the system, each member of which is composed of a collective degree of freedom coupled to a single intrinsic trajectory.

In our numerical simulation, the coupling interaction is not activated at an initial stage. In the beginning, the coupling between the collective and intrinsic systems is switched off, and they evolve independently. Namely, the collective system evolves regularly whereas, as discussed in Sec. III B, the intrinsic system tends to reach its time-independent stationary state (chaotic object). After the statistical state has been realized in the intrinsic system, the coupling interaction is activated. A quantity q_0 in Eq. (43) denotes a value of the collective trajectory q at the switch on time. One purpose of introducing q_0 is to insert the coupling *adiabatically*, and to conserve the total energy before and after the switch on time. (Hereafter, $\tau_{\rm sw}$ denotes the moment when the interaction is switched on, and in our numerical calculation $\tau_{\rm sw}$ is set to be $\tau_{\rm sw}=12\tau_{\rm col}$.)

Here it is worth noting why we let the two systems evolve independently at the initial stage. As is well known, the ergodic and irreversible property of the intrinsic system is assumed in the conventional approach, and the intrinsic system for the *infinite* system is usually represented by the time independent canonical ensemble. In the *finite* system, however, one has to explore whether or not the intrinsic system tends to reach such a state that is effectively replaced by a statistical object, how it evolves after the coupling interaction is switched on, and what its final state looks like.

As discussed at the end of Sec. II B, it is not easy to apply Eq. (25) for analyzing what happens in the dynamical microscopic system, which is in the general situation. Our present primary aim is to microscopically generate such a transport phenomenon that might be understood in terms of the Langevin equation. Namely, we have to construct such a microscopic situation that seems to satisfy Conditions I, II, and III discussed in Sec. II C. In this context, we first let the intrinsic system reach a chaotic situation in a dynamical way, until the ergodic and irreversible property are well realized *dynamically*. In the next subsection, it will be shown that the above microscopic situation is indeed realized *dynamically* for the intrinsic system Eq. (42).

Our attention is mainly focused on examining the energy interchange between these two systems, and the final states these two systems can reach and their interaction dependence. For studying the energy interchange, we make numerical calculations for the following cases. The collective energy is much larger than, comparable to, and much smaller than the intrinsic energy. Namely, the collective energy is chosen to be $E_{\eta}=20$, 40 and 60, whereas the intrinsic energy is fixed at $E_{\xi}=40$. Here $E_{\xi}=40$ is chosen, because the phase space of the intrinsic system is almost covered by the chaotic sea at this energy. In order to examine the interaction dependence of the final state, the interaction strength parameter λ is chosen to be 0.005 (relatively weak), 0.01 and 0.02 (relatively strong).

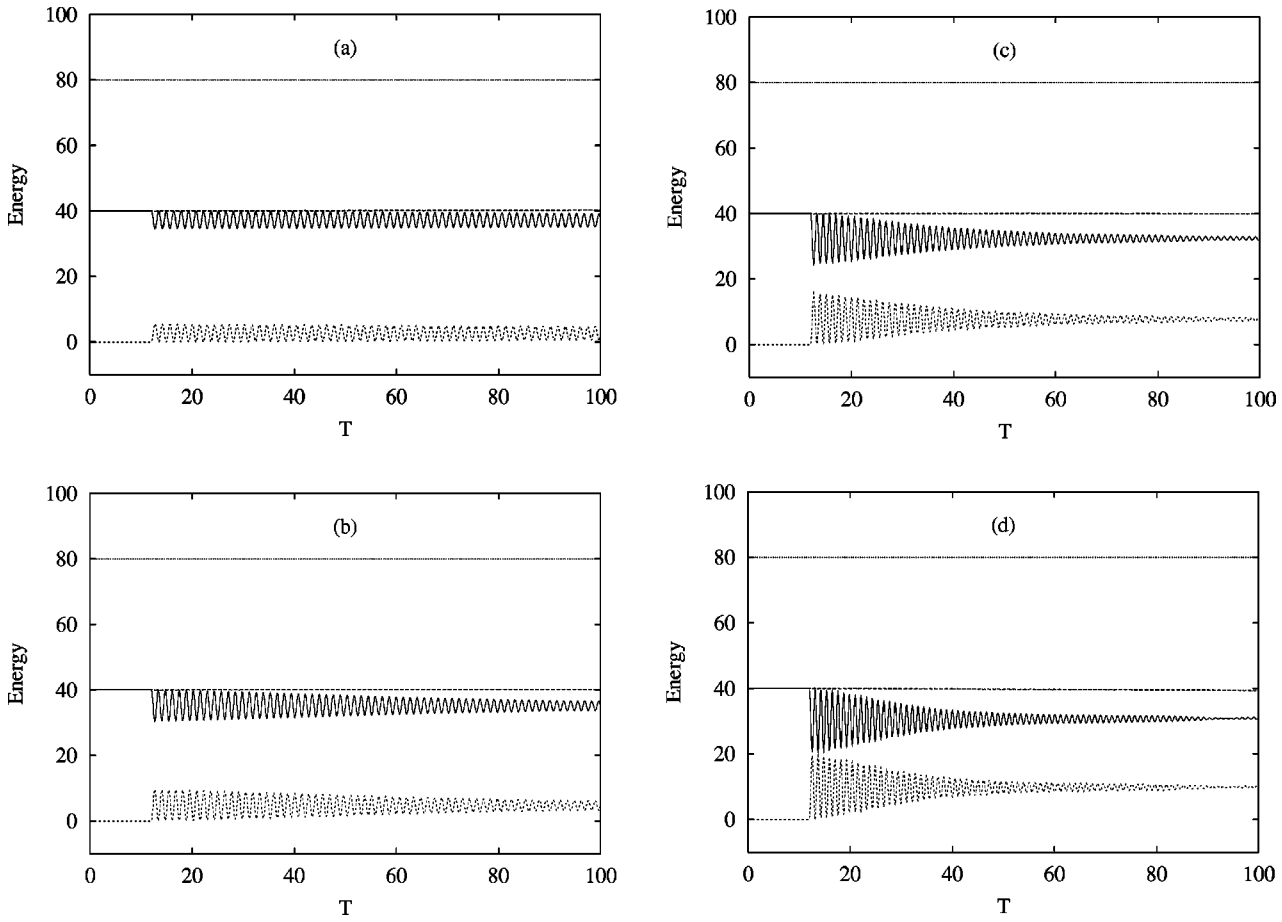

FIG. 1. Time dependence of the averaged partial Hamiltonian $\langle H_\eta \rangle$, $\langle H_\xi \rangle$, $\langle H_{\mathrm{coupl}} \rangle$ and the total Hamiltonian $\langle H \rangle$ for $E_\eta = 40$, $E_\xi = 40$ and (a) $\lambda = 0.005$; (b) $\lambda = 0.01$; (c) $\lambda = 0.02$ and (d) $\lambda = 0.03$. Solid line refers to $\langle H_\eta \rangle$; long dashed line refers to $\langle H_\xi \rangle$; short dashed line refers to $\langle H_{\mathrm{coupl}} \rangle$ and dotted line refers to $\langle H \rangle$. The abscissa T denotes a time in units of τ_{col} which is a characteristic periodic time of the collective oscillator.

B. Energy interchange between the collective and intrinsic systems

Figures 1(a)–1(d) show the time-dependent averaged values of the partial Hamiltonian $\langle H_\eta \rangle$, $\langle H_\xi \rangle$ and $\langle H_{\mathrm{coupl}} \rangle$ and the total Hamiltonian $\langle H \rangle$ defined through

$$\langle X \rangle = \int X \rho(t) dq\, dp \prod_{i=1}^{2} dq_i\, dp_i , \tag{46}$$

for the case with $E_\eta = 40$. One may see that the main change occurs in the collective energy as well as the interaction energy, but not in the intrinsic energy. When one precisely looks for the independent trajectories of the bundle, the collective, intrinsic and interaction energies of each trajectory are changing in time in accordance with the usual Hamilton system. Since the intrinsic system has already reached some stochastic state when the interaction is switched on, a time dependence of the intrinsic energy for each trajectory is canceled out when one takes an average over many trajectories of the bundle. For a case with small interaction strength ($\lambda = 0.005$), the collective energy oscillates for a long time and seems not to reach any saturated value. In the case of relatively large interaction strength ($\lambda \sim 0.02$), it will reach some time-independent value.

Figures 2(a) and 2(b) represent the numerical results for the cases with $E_\eta = 20$ and 60, showing almost the same result as for the case for $E_\eta = 40$. From the above numerical simulation, one may see that the energy is dissipated from the collective to an ''environment,'' when the intrinsic system and the coupling interaction are regarded as an environment. Before understanding the above energy transfer in terms of the phenomenological Langevin equation, it is important to microscopically explore what happens in the intrinsic system when the collective system is attached to the intrinsic system through the coupling interaction.

In Fig. 3, a time dependence of the variance of the intrinsic momentum $\langle p_1^2 \rangle$ is shown. The other intrinsic variances $\langle q_1^2 \rangle$, $\langle q_2^2 \rangle$ and $\langle p_2^2 \rangle$ show almost the same time dependence as in Fig. 3. As discussed in our previous paper [8], an appearance of some chaotic state is expected when the variance has reached its stationary value. Since the variance of the intrinsic system reaches some stationary value before τ_{sw} and since the intrinsic system is regarded as being in the chaotic state, the coupling interaction is activated at τ_{sw} in our simulation. After $\tau_{\mathrm{sw}} = 12\tau_{\mathrm{col}}$, its value remains almost the same for the small interaction strength case, and quickly reaches a little bit larger stationary value for the large coupling strength case ($\lambda = 0.02$). This small increase corresponds to a

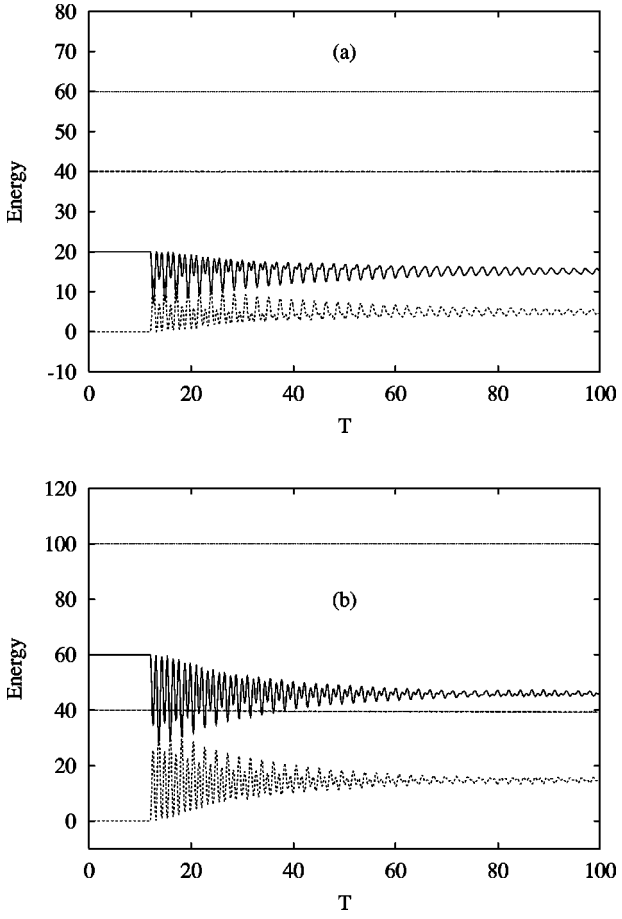

FIG. 2. Time dependence of the averaged partial Hamiltonian for: (a) $E_\eta=20$, $E_\xi=40$, $\lambda=0.02$; (b) $E_\eta=60$, $E_\xi=40$, $\lambda=0.02$. Reference of lines and abscissa T is the same as in Fig. 1.

slight enlargement of the chaotic sea in the intrinsic phase space. Practically, the values of variances are regarded to be constant before and after $\tau_{\rm sw}$.

From our numerical simulation, one may deduce such a conclusion that the intrinsic system even with only two degrees of freedom can be treated as a time independent statistical object before and after the coupling interaction is activated. This conclusion provides us with the dynamical foundation for understanding the statistical ansatz adopted in the conventional transport theory, where the irrelevant system is always regarded as a time-independent statistical object.

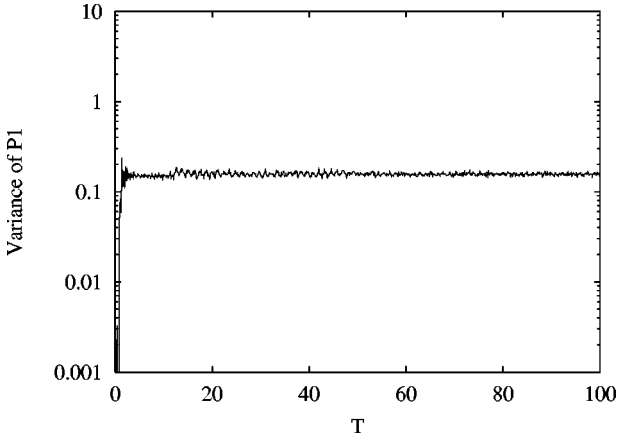

FIG. 3. Time-dependence of variance of p_1 (defined as $\langle p_1^2 - \langle p_1^2\rangle\rangle$) for $E_\eta=40$, $E_\xi=40$ and $\lambda=0.02$. Coupling is switched on at $\tau_{\rm sw}=12\tau_{\rm col}$. Reference of abscissa T is the same as in Fig. 1.

Since the variance has reached its stationary value shortly after $\tau_{\rm sw}$, it is reasonable to introduce the following time independent quantity:

$$\langle p_i^2+q_i^2\rangle=\int\prod_{i=1}^{2}dp_i\,dq_i\{p_i^2+q_i^2\}\rho(t). \tag{47}$$

In accordance with the mean-field Liouvillian in Eq. (20), one may introduce the *time-independent* collective mean-field Hamiltonian as

$$H_\eta+H_\eta(t)|_{t>\tau_{\rm sw}} = \frac{p^2}{2M}+\frac{1}{2}M\omega_0^2q^2+\lambda(q-q_0)^2\sum_{i=1}^{2}\langle p_i^2+q_i^2\rangle. \tag{48}$$

Except for the effects coming from the fluctuation part $H_\Delta(t)$, the collective trajectory is supposed to be described by the mean-field Hamiltonian in Eq. (48) after the coupling interaction is switched on. The solution of Eq. (48) is expressed as

$$q=A\cos\omega(t-\tau_{\rm sw}),\quad p=-M\omega A\sin\omega(t-\tau_{\rm sw}), \tag{49}$$

where

$$\omega^2=\omega_0^2+\omega_1^2,\quad \omega_1^2\equiv\frac{2\lambda}{M}\langle p_i^2+q_i^2\rangle,\quad A=q_0\left(\frac{\omega_0}{\omega}\right)^2, \tag{50}$$

the amplitude A being fixed by using the initial condition $q(\tau_{\rm sw})=q_0$. In accordance with this initial condition, there holds the following energy conservation before and after $\tau_{\rm sw}$ as

$$H_\eta|_{t=\tau_{\rm sw}-0}=H_\eta+H_\eta(t)|_{t=\tau_{\rm sw}+0}=\frac{M}{2}q_0^2\omega_0^2. \tag{51}$$

In order to understand an oscillating property of the collective energy observed in Figs. 1 and 2, let us substitute the solution in Eq. (49) into the collective Hamiltonian H_η. Then one gets

$$H_\eta=\frac{M}{2}q_0^2\omega_0^2\left\{1-4\frac{\omega_1^2\omega_0^2}{\omega^4}\sin^4\frac{\omega}{2}(t-\tau_{\rm sw})\right\}. \tag{52}$$

In Fig. 4, the numerical result of Eq. (52) is shown together with the exact simulated result. As is clearly recognized from Fig. 4 and Eq. (52), the mean-field description can well reproduce the oscillating property (the amplitude, the central energy of the oscillation as well as the frequency)

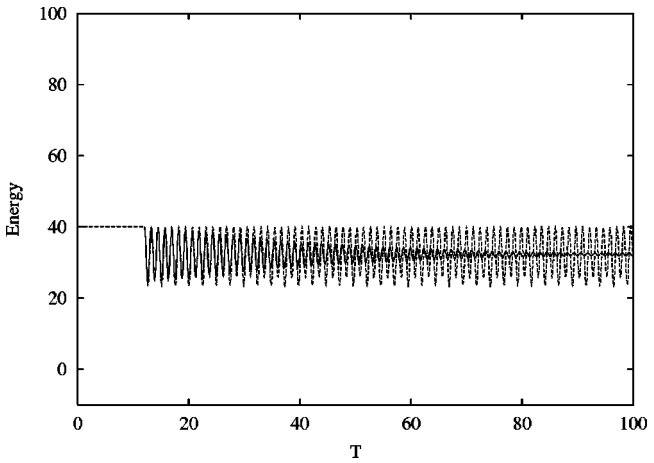

FIG. 4. Time dependence of average collective energy (dashed line) H_η in Eq. (52), in which the mean-field energy of the coupling interaction is considered as shown in Eq. (48), together with the exact simulated result (solid line). Parameters used in the mean-field potential are the same as in Fig. 1(c). Reference of abscissa T is the same as Fig. 1.

of the collective energy $\langle H_\eta \rangle$, whereas it cannot reproduce a reduction mechanism of the amplitude. That is the mean-field Hamiltonian cannot describe the dissipation process. More precisely, one may see that the mean-field approximation provides us with decisive information on the following two points: (a) the amplitude A of the collective energy is determined mainly by the coupling interaction strength λ as well as the averaged properties of the intrinsic system $\langle \Sigma_{i=1}^{2} p_i^2 + q_i^2 \rangle$; (b) the frequency ω is related with the characteristic frequency of the collective oscillator ω_0, the coupling interaction strength λ and the averaged properties of intrinsic system $\langle \Sigma_{i=1}^{2} p_i^2 + q_i^2 \rangle$. From the above discussion and from Figs. 1 and 2, the dissipation process should be attributed to the fluctuation effects coming from H_Δ.

Before discussing the microscopic dynamics responsible for the damping and diffusion process, let us apply the phenomenological transport equation to our present simulated process. Let us suppose that the collective motion will be subject to both a friction force and a random force, and can be described by the Langevin equation. A simple Langevin equation is given by

$$M\ddot{q} + \frac{\partial U^{\mathrm{mf}}(q)}{\partial q} + \gamma \dot{q} = f(t), \tag{53}$$

where $U^{\mathrm{mf}}(q)$ represents the potential part of $H_\eta + H_\eta(t)$ in Eq. (48) and γ the friction strength parameter. A function $f(t)$ represents the random force and, in our calculation, it is taken to be the Gaussian white noise characterized by the following moments:

$$\langle f(t) \rangle = 0, \quad \langle f(t) f(s) \rangle = kT\delta(t-s). \tag{54}$$

The numerical result for Eq. (53) is shown in Fig. 5 with the parameters $\gamma = 0.0033$ and $kT = 1.45$. The parameters appearing in $U^{\mathrm{mf}}(q)$ are the same as in Fig. 1(c).

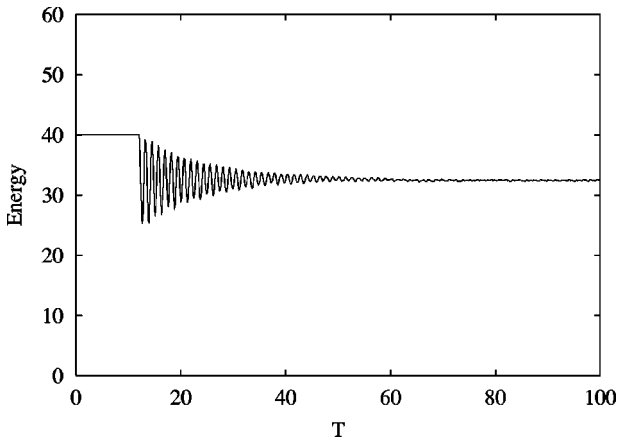

FIG. 5. Time dependence of average collective energy simulated with Langevin equation (53) with $\gamma = 0.0033$ and $kT = 1.45$. Parameters used in the mean-field potential are the same as in Fig. 1(c). Reference of abscissa T is the same as Fig. 1.

As is understood from Fig. 5, the Langevin equation reproduces the energy transfer from the collective system to the environment quite well. This means that our dynamical simulation shown in Fig. 1 is satisfactorily linked with the conventional transport equation, and our schematic model Hamiltonian introduced by Eqs. (40), (42) and (43) is successfully considered as a dynamical *analogue* of the Brownian particle coupled with the classical statistical system. Based on the above analogy and on Eqs. (33) and (53), one may learn the collective degree of freedom is subject to both an average force coming from the mean-field Hamiltonian in Eq. (48) and the fluctuation term H_Δ. Namely, the fluctuation H_Δ described by the last three terms on the right hand side of Eq. (33) is responsible for not only the damping of the oscillation amplitude but also for the dissipative energy flow from the collective system to the environment.

At the end of this subsection, it should be noted that our choice of γ and kT does not satisfy the fluctuation-dissipation theorem. This means that our simulated dissipative phenomenon is not the same as the usual damping phenomena described within the LRT. Since our simulated dissipation phenomenon is induced not by the linear coupling but by the nonlinear coupling, there still remain interesting questions for comprehensively understanding the macroscopic transport phenomena.

C. Microscopic origin of damping and diffusion mechanism

In the Langevin equation, there are two important forces: the friction force and the random force. The former describes the average effect on the collective degree of freedom causing an irreversible dissipation, while the latter describes the diffusion of it. According to the parameter values adopted in our Langevin simulation in Fig. 5, it is naturally expected that the dissipative-diffusion mechanism plays a crucial role in reducing the oscillation amplitude of collective energy, and in realizing the steady energy flow from the collective system to the environment.

In order to explore this point, a time development of the collective distribution function $\rho_\eta(t)$ is shown in Figs. 6 and

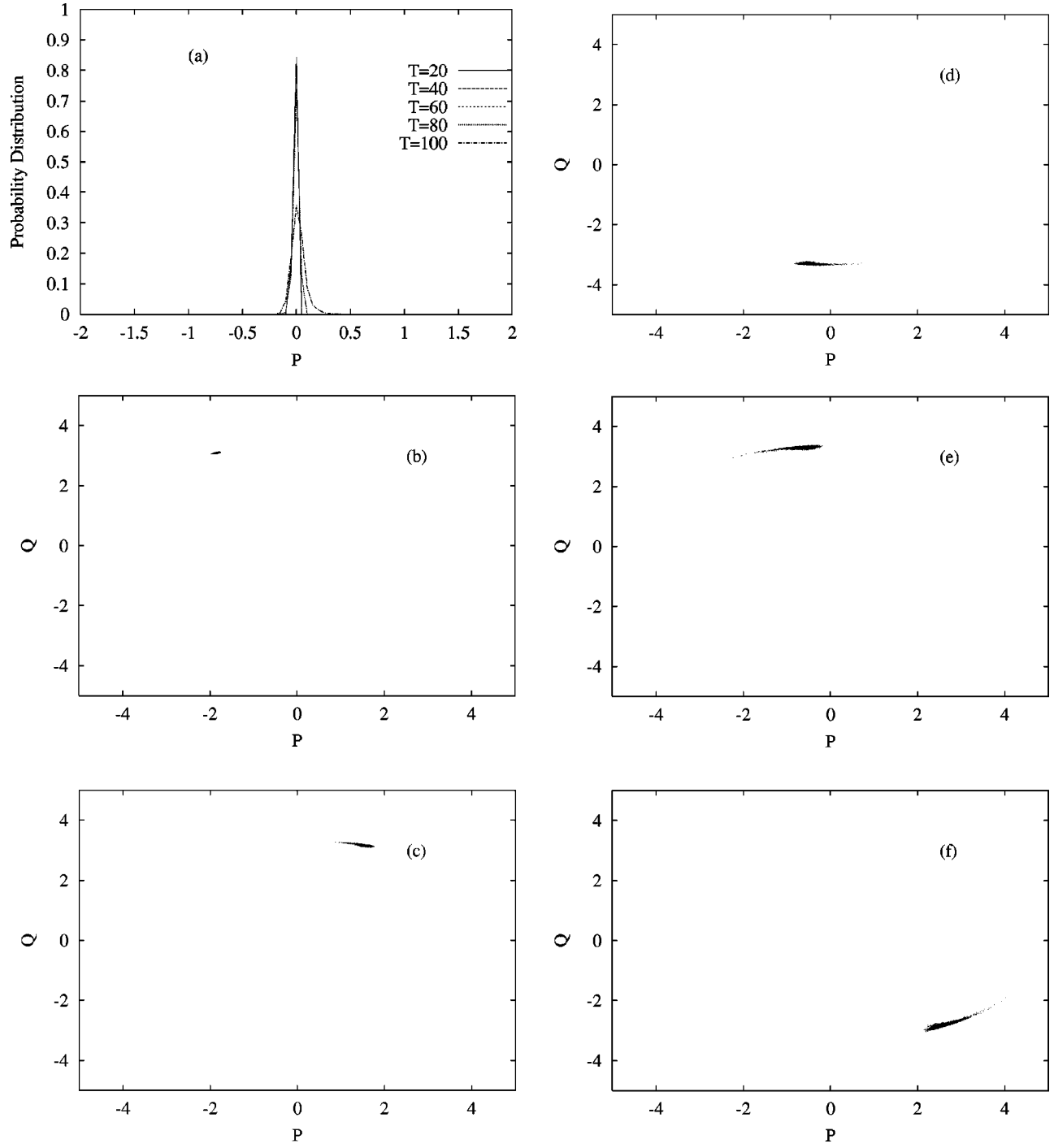

FIG. 6. (a) Probability distribution function of collective trajectories which is defined as $P_\eta(p')=\int \rho_\eta(t)|_{p=p_m+p'}dq$ and p_m satisfies $\partial\rho_\eta(t)/\partial p|_{p=p_m}=0$. The time T is in unit of $\tau_{\rm col}$; (b)–(f) the collective distribution function in (p,q) space at $T=20\tau_{\rm col}$; $T=40\tau_{\rm col}$; $T=60\tau_{\rm col}$; $T=80\tau_{\rm col}$; and $T=100\tau_{\rm col}$ for $E_\eta=40$, $\lambda=0.005$. The parameters are the same as in Fig. 1(c).

7 for two cases with $\lambda=0.005$ (small coupling strength) and 0.02 (large coupling strength). Figures 6(a) and 7(a) illustrate how a shape of the distribution function $\rho_\eta(t)$ in the collective phase space disperses depending on time. In these figures, an effect of the friction force should be observed when a location of the distribution function changes from the outside (higher energy) region to the inside (lower energy) region of the phase space. On the other hand, a dissipative diffusion mechanism is studied from Figs. 6(a) and 7(a) by observing how strongly a distribution function initially (at $t=\tau_{\rm sw}$) centered at one point in the collective phase space disperses depending on time.

One may see that for the case with $\lambda=0.005$, $\rho_\eta(t)$ is slightly enlarged from the initial δ distribution, but is still concentrated in a rather small region even at $t=100\tau_{\rm col}$. On the other hand, for the case with $\lambda=0.02$, one may see that

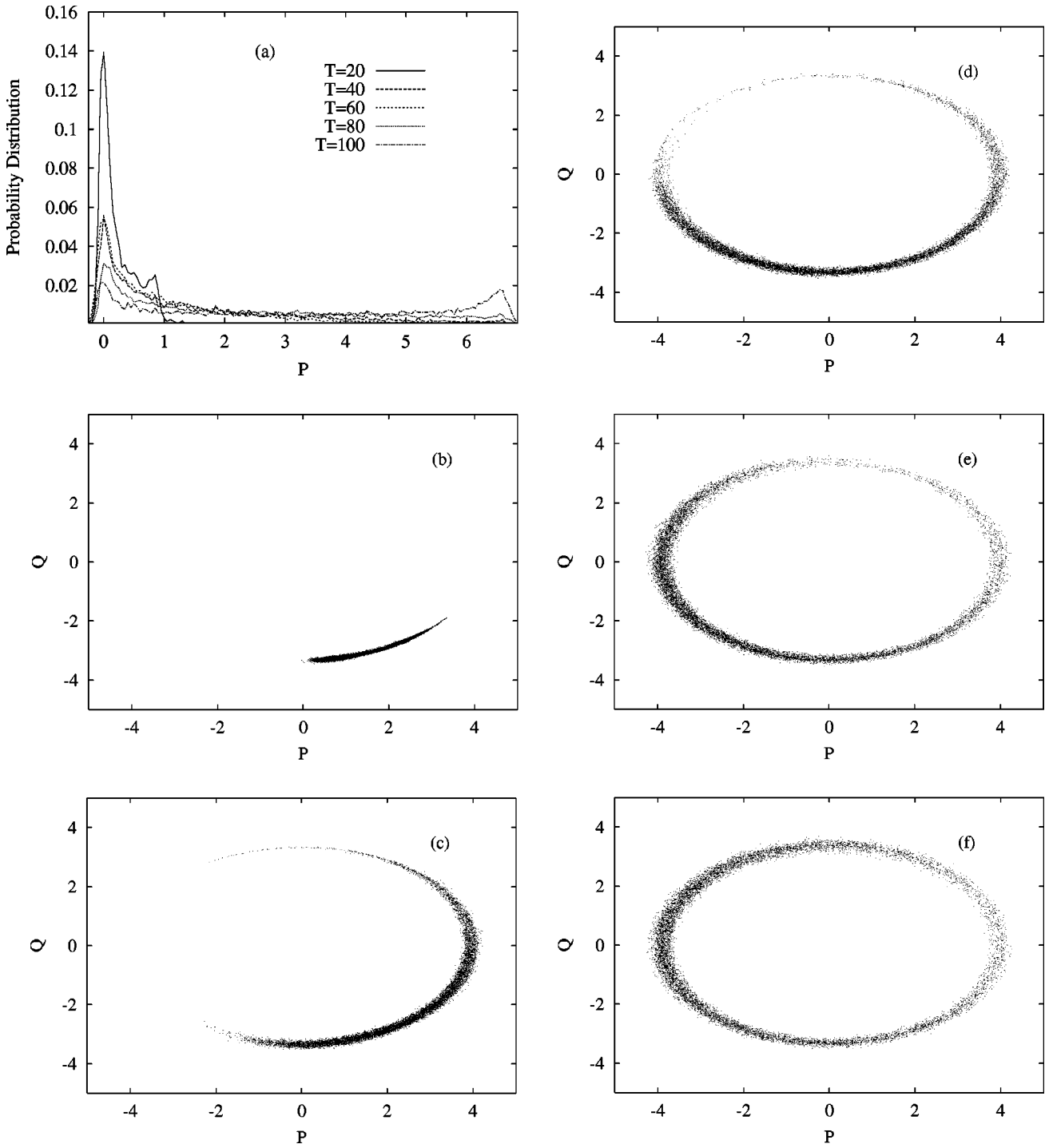

FIG. 7. (a) Probability distribution function of collective trajectories as defined in the caption of Fig. 6(a), (b)–(f) the collective distribution function in (p,q) space at $T=20\tau_{\rm col}$; $T=40\tau_{\rm col}$; $T=60\tau_{\rm col}$; $T=80\tau_{\rm col}$; and $T=100\tau_{\rm col}$ for $E_\eta=40$, $\lambda=0.02$. The parameters are the same as in Fig. 1(c).

$\rho_\eta(t)$ quickly disperses after the coupling interaction is switched on and tends to cover a whole ring shape in the phase space at $t=100\tau_{\rm col}$.

Let us discuss a relation between the reduction mechanism in the amplitude of collective energy and the dispersing property of $\rho_\eta(t)$. Suppose $\rho_\eta(t)$ does not show any strong disperse property by almost keeping its original δ function shape; in this case, the effects coming from $H_\Delta(t)$ are considered to be small. The collective part of each trajectory has a time dependence expressed in Eq. (49) and its collective energy H_η has a time dependence given by Eq. (52). Since there is a well developed *coherence* among the trajectories in $\rho_\eta(t)$ when $\lambda=0.005$, the averaged collective energy $\langle H_\eta\rangle$ over the bundle of trajectories still has a time dependence given by Eq. (52). Consequently, one may not expect a reduction of the oscillation amplitude in the collective energy

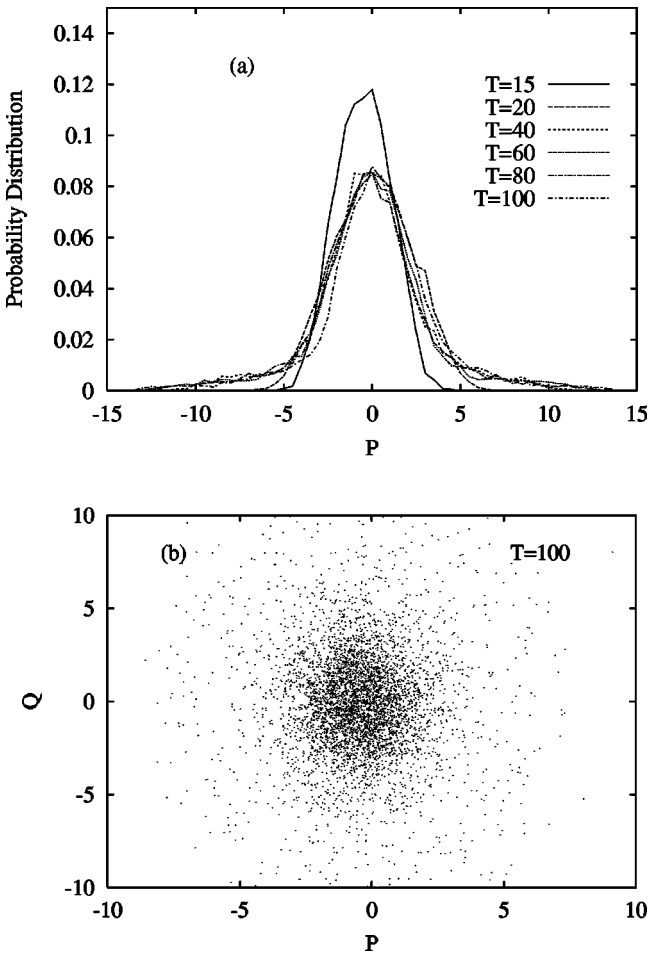

FIG. 8. (a) The probability distribution function of collective trajectories as defined in the caption of Fig. 6(a); (b) collective distribution function in (p,q) space at $t=100\tau_{\mathrm{col}}$ simulated with Langevin equation (53) with $\gamma=0.0033$ and $kT=1.45$. The parameters used in mean-field potential are the same as in Fig. 1(c).

as is shown in Fig. 1(a).

When the distribution function tends to expand over the whole ring shape, the collective part of each trajectory is not expected to have the same time dependence as in Eq. (49). This is due to the effects coming from the stochastic force $H_{\Delta}(t)$, and some trajectories have a chance to have an advanced phase, whereas other trajectories have a retarded phase in comparison with the phase in Eq. (49). According to the *decoherent* effects coming from $H_{\Delta}(t)$, the time dependence of the collective energy for each trajectory in Eq. (49) cancels out due to the randomness of the phases when one takes an average over the bundle of trajectories. This dephasing mechanism is induced by $H_{\Delta}(t)$, and is considered to be the microscopic origin of the damping, i.e., the energy transfer from the collective system to the environment.

In order to compare the above mechanism with what happens in the phenomenological transport equation, the solution of the Langevin equation represented in the collective phase space is shown in Fig. 8 for the cases with $\gamma=0.0033$ and $kT=1.45$. From this figure, one may understand that the damping (a change of the distribution from the outside to the inside of the phase space) as well as the diffusion (an expansion of the distribution) are taking place so as to reproduce the numerical result in Fig. 5. Even though the Langevin equation gives almost the same result as in Fig. 1 in the macroscopic level, as is recognized by comparing Figs. 6 and 7 with Fig. 8, there are substantial differences in the microscopic-level dynamics. Namely, the distribution function $\rho_{\eta}(t)$ of our simulation evolves into the whole ring shape while retaining almost the same initial energy region of the phase space, while the solution of the Langevin equation evolves to a round shape while covering the whole energetically allowed region. In the case of the Langevin simulation, the dissipation and dephasing mechanisms seem to help reproduce the result in Fig. 5, while the dephasing mechanism is essential for the damping of the collective energy in our microscopic simulation.

D. Linear and nonlinear coupling

According to the SCC method, which has been developed to optimally divide the total space into the relevant and irrelevant subspaces, there should not be any linear coupling interaction between the two spaces. In other words, one may optimally divide the total system into the two *decoupled* subsystems by using such a dynamical condition that the linear coupling between them should be eliminated. Since a ratio between the time scale of the well developed collective motion and that of the single-particle motion is typically less than 1 order of magnitude in such a finite system as the nucleus, it is a very important task to carefully study how the relevant degrees of freedom are distinguished from the remaining degrees of freedom. On the basis of the SCC method, one may state that the separation of the total system into two subsystems coupled with a linear interaction has no physical meaning in a finite system, because a choice of the coordinate system, i.e., a separation between the relevant and irrelevant coordinates, remains arbitrary when a linear coupling remains between them. This statement is easily recognized when one remembers that the harmonic oscillators coupled with the linear interaction reduce to the uncoupled harmonic oscillators by a proper choice of the coordinate system. Here, we do not intend to extend the above statement for the infinite system, because there is a many order of magnitude difference between a time scale of the macroscopic motion and that of the microscopic one, and there are huge numbers of degrees of freedom in the infinite irrelevant system.

In order to explore the different effects between the linear and nonlinear coupling interactions on the dissipative process, we have made a numerical simulation for the β-FPU model described in Refs. [12,21–23]. The collective H_{η}, intrinsic H_{ξ} and coupling H_{coupl} Hamiltonians are given as

$$H_{\eta}=\frac{p^2}{2}+\frac{\omega^2 q^2}{2}, \quad p=\dot{q},$$

$$H_{\xi}=\sum_{i=1}^{N}\frac{p_i^2}{2}+\sum_{i=2}^{N}W(q_i-q_{i-1})+W(q_N), \tag{55}$$

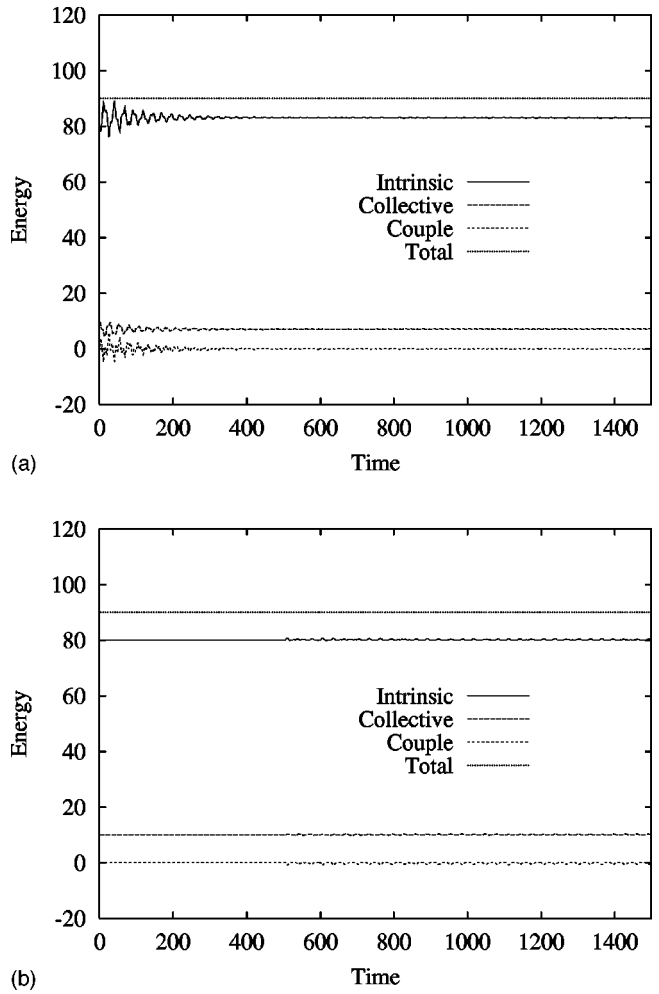

FIG. 9. Distribution of the partial Hamiltonian $\langle H_\eta \rangle$, $\langle H_\xi \rangle$, $\langle H_{\rm coupl} \rangle$, and $\langle H \rangle$ for the system described as Eq. (55).

$$W(q) = \frac{q^4}{4} + \frac{q^2}{2}, \quad p_i = \dot{q}_i,$$

$$H_{\rm coupl} = \Delta q q_1 .$$

The parameters used in our numerical calculation are $N=8$, $\omega = 0.2$, and $\Delta = 0.02$. A number of pseudo-particle is 10 000. As an initial condition for each pseudo-particle, we take each oscillator energy $\epsilon = 10$ with $q_i^{(0)} = 0$ and $q^{(0)} = 0$.

The numerical results are illustrated in Fig. 9. In Fig. 9(a), the coupling is switched on from the beginning, whereas in Fig. 9(b) it is switched on at $\tau_{\rm sw} = 600\tau_{\rm in}$, when a chaotic situation has been well realized in the intrinsic system. In Fig. 9(a), one may observe small energy transfer from the collective to the intrinsic system when the system reach its stationary state at $t \approx 400\tau_{\rm in}$. Namely $\langle H_\xi \rangle$ becomes a little bit greater than 80 and $\langle H_\eta \rangle$ less than 10. Before reaching their stationary states, especially at the early stage at $t \leqslant 100\tau_{\rm in}$, there is a violent energy exchange between the collective and intrinsic systems. Since it is not allowed to apply any statistical treatment for the intrinsic system in this early stage when no stationary state is realized, there might be no reason to apply the Langevin type equation for a case in Fig. 9(a). In other words, the above energy transfer may not be understood in macroscopic terms. When one switches on the coupling after the chaotic state has been realized in the intrinsic system, there is almost no energy dissipation in the collective motion as is seen from Fig. 9(b).

An essential difference between the linear and nonlinear coupling cases may be understood as follows: As is seen from Eq. (48), the coupling $H_{\rm coupl}$ produces the mean-field potential $H_\eta(t)$ in the case of the nonlinear coupling, because the second moment $\langle \Sigma_{i=1}^2 \{q_i^2 + p_i^2\} \rangle$ has some value when the intrinsic system reaches some stationary state. It is recognized from Eq. (52) that this average effect plays a decisive role in defining an amount of transferred energy from the collective system to the environment, like the friction force. On the other hand, $H_{\rm coupl}$ does not produce any averaged effects on the collective motion in the case of linear coupling, because there is a relation $\langle q_i \rangle = 0$ when the statistical state is realized in the intrinsic system. With regards to the β-FPU model, one may conclude that the energy dissipation phenomena may not be expected, although the other main numerical results described in Ref. [12] have been reproduced.

IV. CONCLUDING REMARKS

In the present paper, the transport phenomenon in a finite system is *dynamically* established for the first time. It is shown that the optimal separation between the relevant and irrelevant degrees of freedom performed by the self-consistent collective coordinate (SCC) method, which dynamically eliminates the linear coupling and leaves the nonlinear coupling between two subsystems, is decisive for generating the transport phenomenon in the finite system. To realize the transport phenomenon capable of being studied within the fully microscopic coupled master equation (25), where the coupling between the relevant and irrelevant systems is divided into the averaged (mean-field) part and the fluctuation part, we concentrate on a specific case where the relevant and irrelevant systems are evolved independently at the initial stage for the purpose of generating such a physical situation where Conditions I, II, and III are satisfied as discussed in Sec. II C.

With the aid of numerical simulation, it has been clarified that the microscopic dephasing mechanism, which is caused by the chaoticity of the irrelevant system, is responsible for the energy transfer from the collective system to the environment. The established transport phenomenon is successfully reproduced by the Langevin equation, whose potential $U^{\rm mf}$ is determined by the mean field collective Hamiltonian $H_\eta + H_\eta(t)$. Even in this specific case, it has been clarified that there are substantial differences in the micro-level mechanism between the full microscopic description and the Langevin description, although both descriptions well describe the same macro-level transport phenomenon.

From our present work, one may conclude that there still remains a number of subjects, such as: (a) What is the effect of the number of degrees of freedom of irrelevant system; (b) What will happen in the case where the intrinsic system is in the mixed situation of chaotic and regular motion, since its

TDHF manifold has a very rich structure as shown in our previous paper [8]? (c) Since there might be other various mechanisms responsible for the dissipation process, besides the dephasing mechanism discussed in the present paper, a more general form of coupling interaction should be considered; (d) In LRT [13], an appearance of macroscopic damping is intimately related to the introduction of a finite width of the single-particle level or the continuum distribution of spectrum frequency. In the nonlinear interaction case, we have shown that the dissipative behavior is realized without introducing such prescriptions. So it is worth clarifying whether or not the continuum distribution of spectrum frequency can be obtained self-consistently in the case of nonlinear interaction. The above-mentioned works are in progress.

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